Examiners' Report: Final Honour School of Mathematics Part C Trinity Term 2021

January 24, 2022

Part I

A. STATISTICS

- **Numbers and percentages in each class**

  See Table 1, page 1.

- **Numbers of vivas and effects of vivas on classes of result.**
  As in previous years there were no vivas conducted for the FHS of Mathematics Part C.

- **Marking of scripts.**
  The dissertations and mini-projects were double marked. The remaining scripts were all single marked according to a pre-agreed marking scheme which was very closely adhered to. For details of the extensive checking process, see Part II, Section A1.

- **Numbers taking each paper.**
  See Table 7 on page 11.

Table 1: Numbers in each class (new MSc classification)

	Number	Percentages %
	2021	2021
Distinction	60	60
Merit	20	20
Pass	18	18
Fail	2	2
Total	100	100

Table 2: Numbers in each class (pre-2021 classification)

	Number					Percentages %				
	2020	(2019)	(2018)	(2017)	(2016)	2020	(2019)	(2018)	(2017)	(2016)
I	63	(58)	(53)	(48)	(44)	67.74	(57.43)	(56.99)	(57.14)	(50.57)
II.1	30	(40)	(26)	(23)	(31)	32.26	(39.6)	(27.96)	(27.38)	(35.63)
II.2	0	(2)	(13)	(12)	(9)	0	(1.98)	(13.98)	(14.29)	(10.34)
III	0	(1)	(1)	(1)	(3)	0	(0.99)	(1.08)	(1.19)	(3.45)
F	0	(0)	(0)	(0)	(0)	0	(0)	(0)	(0)	(0)
Total	93	(101)	(93)	(84)	(87)	100	(100)	(100)	(100)	(100)

B. New examining methods and procedure in the 2020 examinations

In light of the ongoing Covid 19 pandemic, the University changed the examinations to an open-book format and rolled out a new online examinations platform. An additional 30 minutes was added on to the exam duration to allow candidate the technical time to download and submit their examination papers via Inspera.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

The department intends to hold in person, partially open book exams in Trinity Term 2022: students will be permitted to bring a couple of pages of notes into the exam, prepared by the student, with adjustments in cases of disability.

D. Notice of examination conventions for candidates

The first notice to candidates was issued on 26th February 2021 and the second notice on 30th April 2021. These contain details of the examinations and assessments.

All notices and the examination conventions for 2021 examinations are on-line at http://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments.

Part II

A1. General Comments on the Examination

The examiners would like to convey their grateful thanks for their help and cooperation to all those who assisted with this year's examination, either as assessors or in an administrative capacity. The chairman would like to thank Barbara Galinska, Charlotte Turner-Smith,

Waldemar Schlackow and the rest of the academic administration team for their support of the Part C and OMMS examinations.

In addition the internal examiners would like to express their gratitude to Prof Richard Jozsa and Prof James Robinson for carrying out their duties as external examiners in a constructive and supportive way during the year, and for their valuable input at the final examiners' meetings.

Timetable

The examinations began on Monday 31st May and finished on Friday 18th June.

Mitigating Circumstances Notice to Examiners and other special circumstances

A subset of the examiners (the 'Mitigating Circumstances Panel') attended a pre-board meeting to band the seriousness of the individual notices to examiners. The outcome of this meeting was relayed to the Examiners at the final exam board, who gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate. See Section E for further details.

Setting and checking of papers and marks processing

Following established practice, the questions for each paper were initially set by the course lecturer, with the lecturer of a related course involved as checker before the first draft of the questions was presented to the examiners. The course lecturers also acted as assessors, marking the questions on their course(s).

The internal examiners met in early January to consider the questions on Michaelmas Term courses, and changes and corrections were agreed with the lecturers where necessary. The revised questions were then sent to the external examiners. Feedback from external examiners was given to examiners, and to the relevant assessor for each paper for a response. The internal examiners met a second time late in Hilary Term to consider the external examiners' comments and assessor responses (and also Michaelmas Term course papers submitted late). The cycle was repeated for the Hilary Term courses, with the examiners' meetings in the Easter Vacation. Before questions were submitted to the Examination Schools, setters were required to sign off a camera-ready copy of their questions.

Candidates accessed and downloaded their exam papers via the Inspera system at the designated exam time. Exam responses were uploaded to Inspera and made available to the Exam Board Administrator 25-33.5 hours after the exam paper had finished via One Drive.

The process for Marking, marks processing and checking was adjusted accordingly to fit in with the online exam responses. Assessors had a week to return the marks on the mark sheets provided. A check-sum was also carried out to ensure that marks entered into the database were correctly read and transposed from the mark sheets.

All scripts and completed mark sheets were returned, if not by the agreed due dates, then at least in time for the script-checking process.

A team of graduate checkers, under the supervision of Barbara Galinska, reviewed the mark sheets for each paper of this examination, carefully cross checking against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. Also sub-totals for each part were checked against the mark scheme, noting correct addition. In this way a number of errors were corrected, each change was approved by one of the examiners who were present throughout the process.

Determination of University Standardised Marks

The Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average in each class over the last four years, together with recent historic data for Part C, the MPLS Divisional averages, and the distribution of classifications achieved by the same group of students at Part B.

The examiners followed established practice in determining the University standardised marks (USMs) reported to candidates. This leads to classifications awarded at Part C broadly reflecting the overall distribution of classifications which had been achieved the previous year by the same students.

We outline the principles of the calibration method.

The Department's algorithm to assign USMs in Part C was used in the same way as last year for each unit assessed by means of a traditional written examination. Papers for which USMs are directly assigned by the markers or provided by another board of examiners are excluded from consideration. Calibration uses data on the Part B classification of candidates in Mathematics and Mathematics & Statistics (Mathematics & Computer Science and Mathematics & Philosophy students are excluded at this stage). Working with the data for this population, numbers N_1, N_2 and N_3 are first computed for each paper: N_1, N_2 and N_3 are, respectively, the number of candidates taking the paper who achieved in Part B overall average USMs in the ranges $[70, 100]$, $[60, 69]$ and $[0, 59]$, respectively.

The algorithm converts raw marks to USMs for each paper separately (in each case, the raw marks are initially out of 50, but are scaled to marks out of 100). For each paper, the algorithm sets up a map $R \to U$ (R = raw, U = USM) which is piecewise linear. The graph of the map consists of four line segments: by default these join the points $(100, 100)$, $P_1 = (C_1, 72)$, $P_2 = (C_2, 57)$, $P_3 = (C_3, 37)$, and $(0, 0)$. The values of C_1 and C_2 are set by the requirement that the proportion of I and II.1 candidates in Part B, as given by N_1 and N_2, is the same as the I and II.1 proportion of USMs achieved on the paper. The value of C_3 is set by the requirement that P_2P_3 continued would intersect the U axis at $U_0 = 10$. Here the default choice of *corners* is given by U-values of 72, 57 and 37 to avoid distorting nonlinearity at the class borderlines.

The results of the algorithm with the default settings of the parameters provide the starting point for the determination of USMs. The examiners have scope to make changes, usually by adjusting the position of the corner points P_1, P_2, P_3 by hand, so as to alter the map raw $\to$ USM, to remedy any perceived unfairness introduced by the algorithm, in particular in cases where the number of candidates is small. They also have the option to introduce additional corners.

Table 3 on page 5 gives the final positions of the corners of the piecewise linear maps used to determine USMs from raw marks. For each paper, P_1, P_2, P_3 are the (possibly adjusted) positions of the corners above, which together with the end points $(100, 100)$ and $(0, 0)$ determine the piecewise linear map raw $\rightarrow$ USM. The entries N_1, N_2, N_3 give the number of incoming firsts, II.1s, and II.2s and below respectively from Part B for that paper, which are used by the algorithm to determine the positions of P_1, P_2, P_3.

Following customary practice, a preliminary, non-plenary, meeting of examiners was held two days ahead of the plenary examiners' meeting to assess the results produced by the algorithm alongside the reports from assessors. The examiners reviewed each papers and report, considered whether open book examination process affected candidates and reviewed last year's stats. The examiners discussed the preliminary scaling maps and the preliminary class percentage figures. Adjustments were made to the default settings as appropriate, paying particular attention to borderlines and to raw marks which were either very high or very low. These revised USM maps provided the starting point for a review of the scalings, paper by paper, by the full board of examiners.

Table 3: Position of corners of piecewise linear function

Paper	P_1	P_2	P_3	Additional corners	N_1	N_2	N_3
C1.1	20;50	30;60	37;70	50;100	2	0	0
C1.2	4.14;37	10;50	24;60	40;70	2	2	0
C1.3	8.73;37	17;57	30;70	50;100	8	5	0
C1.4	8;50	13;60	24;70	50;100	4	4	0
C2.1	24;50	30;60	37;70	50;100	13	5	0
C2.2	4.54;37	21;50	26;57	35;72	11	9	0
C2.3	9.48;37	23;60	29;70	50;100	7	2	0
C2.4	16;50	20;60	35.2;72	50;100	6	5	0
C2.5	9.10;37	20;57	28;72	50;100	5	2	0
C2.6	12.93;37	21;50	27;60	35;70	7	2	0
C2.7	18;50	24;60	32;70	50;100	17	11	0
C3.1	12;50	20;60	31;72	50;100	10	5	0
C3.2	5.51;37	9.6;57	36.6;72	50;100	4	5	0
C3.3	13;50	20.1;57	24;70	50;100	6	1	0
C3.4	8.56;37	25;50	30;60	40.4;72	8	4	0
C3.5	8.85;37	21;57	33.4;72	50;100	5	3	0
C3.7	7.07;37	18;50	36;70	50;100	11	6	0
C3.8	6;30	18;50	25;60	41;72	10	10	0
C3.10	13;50	21;60	32;70	50;100	3	3	0
C3.11	19;61	25;70	50;100	-	3	0	0
C4.1	12.52;37	17;55	30;70	50;100	7	2	0
C4.3	15;50	27;60	35;70	50;100	9	3	0
C4.6	6.43;37	20;50	30;60	35;70	7	2	0
C4.8	15;50	23;60	30;70	50;100	1	0	0
C4.9	22;50	27;60	32;70	50;100	2	1	0

Paper	P_1	P_2	P_3	Additional corners	N_1	N_2	N_3
C5.1	13.79;37	28;60	39;72	50;100	7	6	1
C5.2	5.80;37	22;60	29.6;72	50;100	9	6	1
C5.3	15.80;37	30;60	40;70	50;100	2	3	0
C5.5	10.40;37	20;50	29;60	37.6;72	14	15	3
C5.6	14;50	30;60	42;70	50;100	4	6	1
C5.7	8.85;37	25;60	30;70	50;100	8	9	1
C5.9	20;50	28;60	36;70	50;100	5	2	1
C5.11	13.39;37	23.3;57	25;60	36.8;72	13	12	2
C5.12	13.79;37	35;70	50;100	-	10	14	1
C6.1	7.41;37	21;60	32;70	50;100	7	15	4
C6.2	13.16;37	22;50	27;60	33.4;72	4	11	1
C6.3	11.78;37	20.5;57	34;70	50;100	1	5	1
C6.4	21;50	30;60	35;70	50;100	6	4	0
C7.4	22;50	35;60	39;70	50;100	2	2	1
C7.5	15;50	22;60	32;70	50;100	1	1	0
C7.6	17;50	23;60	30;70	50;100	1	0	0
C7.7	24;50	29;60	36;70	50;100	2	4	2
C8.1	25;50	32;60	42;70	50;100	7	9	0
C8.2	17;50	32.6;72	50;100	-	5	5	0
C8.3	14;50	30;60	35;70	50;100	10	12	1
C8.4	12;50	18;66	24;72	50;100	7	7	1
C8.6	14.88;37	25.9;57	36.4;72	50;100	3	3	0
SC1	9.54;37	30;60	38;70	50;100	13	13	1
SC2	13.961;37	24.3;57	37.8;72	50;100	14	16	4
SC4	11.83;37	25;60	32.6;72	50;100	10	12	2
SC5	20;40	35;60	45;70	50;100	9	10	2
SC6	13.96;37	20;50	35;70	50;100	0	4	3
SC7	15;40	25;60	34;70	50;100	5	4	1
SC9	0;0	7.87;37	22;60	30.2;72	6	7	0
SC10	6.03;37	25;60	33;72	50;100	7	4	1

A2. Issues to do with the Inspera computer system

It was decided at the University level that mathematics online, open book exams would happen through the Inspera computer system. This was run by an IT team at the University level, and Mathematical Institute staff had no control over it.

The Inspera system has some features that are unsuitable for the needs of mathematics exams. The first issue we discuss was obvious to us from the outset, and Mathematical Institute staff protested about it early on, but no changes were made.

(a) Candidates locked out of Inspera immediately at end of exam

The way the system actually worked: If candidates go past their submission deadline, the Inspera system locks them out, and they are unable to submit their script through Inspera. So they then had to e-mail their script to Maths Institute staff.

How we wanted the system to work: It would have been immensely helpful if the Inspera system could have accepted late submissions, flag them as late, and record the submission time. This does not sound like a difficult feature to program in.

It seems that Inspera is designed primarily around the idea that candidates type their responses into Inspera continuously during the exam, and at the end of the exam the system closes and no more input is accepted.

Mathematics undergraduates will overwhelmingly want to write their scripts by hand, usually on paper, to accommodate diagrams, equations, and mathematical symbols not on QWERTY keyboards; to spend any spare time correcting solutions or solving remaining parts, until the last minute; and to scan and upload their scripts only at the very end.

Predictably, a significant number of candidates went past their submission deadline by a few minutes because of last minute problems with bad wifi / problems scanning scripts / phone or computer battery problems / the Inspera site itself misbehaving, and were locked out of Inspera. Processing these late submissions and the MCEs submitted by candidates took up a lot of administrative time, and caused the candidates anxiety.

When they were locked out of Inspera, some candidates contacted their college, waited for a reply, and only e-mailed us their script 1-2 hours later. We have no way to know whether they did further work on their answers in that time. The examiners chose not to penalize any candidate with a reasonable excuse (easy to come up with) for late submission.

Recommendations for future online exams: *please* can the Inspera/other system accept late submissions and record how late they are, this would be so much easier.

Also, please could the penalties for a short time overrun be mild, not draconian, e.g. deduct 1 USM for 0-5 minutes overrun, and we should announce our intention of applying these mild penalties even to people with plausible excuses, giving them an incentive to go somewhere with good wifi, and charge their phones in advance.

The trouble with draconian penalties for minor infractions (e.g. submitting 6 minutes late = fail paper, hence also final class capped at pass, as in this year's Examination Conventions) is that the examiners are very unwilling to apply them, so we waive them. At the same time, we guess some short overruns may happen like this: a student calculates that they can scan and upload their script in 20 minutes, so they take 10 minutes of the 30 minutes "technical time" to work on their script. Then some minor problem occurs, as in the list above, and scanning and uploading takes 26 minutes.

(b) Wrong exam papers made available on Inspera

For two exams, C2.6 and C6.1, the Inspera IT team initially made the wrong exam paper available for download. For C2.6, only the cover page was available, without the questions. For C6.1, they actually made a different exam available, C6.2, which was to be sat 4 days later. This compromised the security of C6.2, so as instructed by the proctors, we had to set and check an entire new C6.2 exam in 3 days. The examiners are very grateful to the C6.2 assessor, who sacrificed much of her weekend time with her family to make this happen.

Five candidates submitted MCEs saying that downloading the wrong exam had caused them panic attacks or similar during the exam.

The exams were correct when submitted to the Exam Schools, and we believe the mistakes were introduced by the Inspera IT team. We appreciate that Maths exams are unlikely to be comprehensible to a non-specialist. However, even very basic error checks might have uncovered the fact that an exam paper had no questions, or had a different paper number and completely different title. Producing a cover page only is presumably the result of running LaTeX on the paper with only half the source files. This would have spat out a list of fatal errors about missing files, it is a pity these were not acted on.

(c) Response of Inspera IT team to making wrong exam available

How the Inspera IT team responded to helpline calls saying that the C2.6 and C6.1 exam papers were wrong: they e-mailed the correct exam paper, simultaneously, to all students taking the paper, whether or not the students had started the exam and downloaded the incorrect paper. For those that had started the paper, they also reset their clock to a later starting time the Inspera team chose.

This had several adverse consequences:

- We have students in multiple time zones, who were expecting to start their exams up to 17 hours later than the standard UK start time of 9.30am. So a number of students were e-mailed the exam up to 16-17 hours before they actually started. If they opened the e-mail, they could have spent much longer on their solutions. Comparing 'early' candidates' USMs with their performance on other papers suggests that some candidates benefitted substantially from this.
- Even students in the UK can choose their start time in a 24 hour window. So candidates that had not started the exam before the paper was e-mailed could have chosen to delay their start time and spend extra time on the exam. We have no evidence any candidates did this, but it was possible.
- Some candidates who had their clocks restarted by the Inspera team submitted MCEs to say that they did not open the e-mail until after their clock had restarted, and so they lost time to do the exam.

The examiners were careful to choose the scaling maps so that candidates who did not receive the exam early were not disadvantaged (that is, we guarded against the possibility that 'early' candidates may have received higher marks, increasing the average score, resulting in the algorithm delivering a lower scaling function, decreasing the USMs of non-'early' candidates). However, we did not penalize in any way the candidates who were e-mailed the exam before they started it, and these very well may have gained an advantage.

How we would have preferred the Inspera IT team to respond: it would have been much better not to e-mail the paper to all candidates, but to reset the Inspera system with the correct paper, and allow all candidates to start the exam and download the paper at a time of their choosing within a 24 hour window, and e-mail all candidates to tell them this.

We do not know why the Inspera team did not do this. But as it is so obvious, we guess that some technical feature of the Inspera system made it impractical. If so, we recommend that the appropriate functionality should be added to the Inspera system.

(d) General comments

The issues discussed above had significant negative consequences for the fairness of the exams this year, and for the mental well being of students and staff, and caused a lot of unnecessary extra work for Maths Institute staff during a very busy time.

If the system (or a similar one) is to be used again in a future year, we **recommend** that the functionality of the system be reviewed well in advance to ensure that it is fit for purpose, bearing in mind the particular specialist needs of subjects such as mathematics.

We also **recommend** that procedures within the Inspera IT department for handling papers to avoid introducing errors, and to detect such errors, and for what to do if a problem arises with a paper once it is released to students, should all be carefully reviewed.

B. Equality and Diversity issues and breakdown of the results by gender

Table 5: Breakdown of results by gender

Class	Number		
	2021		
	Female	Male	Total
Distinction	15	45	60
Merit	8	12	20
Pass	5	13	18
Fail	1	1	2
Total	29	71	100
Class	Percentage		
	2021		
	Female	Male	Total
Distinction	51.72	63.38	60
Merit	27.59	16.9	20
Pass	17.24	18.31	18
Fail	3.45	1.41	2
Total	100	100	100

Table 6: Breakdown of results by gender

Class	Number								
	2020			2019			2018		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	16	47	63	8	50	58	6	47	53
II.1	4	26	30	9	31	40	7	19	26
II.2	0	0	0	0	2	2	3	10	13
III	0	0	0	0	1	1	1	0	1
F	0	0	0	0	0	0	0	0	0
Total	20	73	93	17	84	101	17	76	93
Class	Percentage								
	2020			2019			2018		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
I	80	64.38	72.19	47.06	59.52	57.43	35.29	61.84	56.99
II.1	20	35.62	27.81	52.94	36.9	39.6	41.18	25	27.96
II.2	0	0	0	0	2.38	1.98	17.65	13.16	13.98
III	0	0	0	0	1.19	0.99	5.88	0	1.08
F	0	0	0	0	0	0	0	0	0
Total	100	100	100	100	100	100	100	100	100

C. Detailed numbers on candidates' performance in each part of the exam

Data for papers with fewer than six candidates are not included.

Table 7: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM
C1.1	-	-	-	-	-
C1.2	-	-	-	-	-
C1.3	13	28.54	9.25	70.38	12.08
C1.4	8	21.62	8.8	66.5	13.19
C2.1	18	35.28	9.07	68.94	17.49
C2.2	20	31.15	8.84	66.7	13.97
C2.3	9	31.33	9.67	72.67	14.87
C2.4	11	30.18	8.74	69	10.31
C2.5	7	24.29	6.26	64.43	10.86
C2.6	9	38.22	8.66	78.33	14.48
C2.7	28	30.61	7.67	68.36	12.78
C3.1	15	31.13	11.63	72.93	17.72
C3.2	9	31	8.83	70.44	8.53
C3.3	7	27.86	4.1	73.43	6.73
C3.4	12	37.75	8.78	72.5	15.81
C3.5	8	30.12	7.45	68.38	9.97
C3.7	17	36.76	8.62	75	13.21
C3.8	19	33.16	12.28	69.11	18.2
C3.10	6	25.5	18.01	56	35.19
C3.11	-	-	-	-	-
C4.1	8	32.62	7.07	74.75	9.79
C4.3	12	33.17	6.85	69.25	9.31
C4.6	9	35.11	9.25	71.78	15.18
C4.8	-	-	-	-	-
C4.9	-	-	-	-	-
C5.1	14	34.36	9.25	68.71	14.71
C5.2	16	26.25	6.58	66.75	9.71
C5.3	-	-	-	-	-
C5.5	31	33.52	7.54	67.71	11.59
C5.6	11	38.64	10.22	73.27	15.75
C5.7	18	29.72	5.99	68.94	9.97
C5.9	8	36.88	8.92	74.88	15.46
C5.11	25	33.84	6.65	70.48	9.75
C5.12	24	33.88	5.47	69.12	9.43
C6.1	17	26.59	9.09	66.24	12.27
C6.2	15	30.07	3.37	65.67	6.31
C6.3	-	-	-	-	-
C6.4	10	34.2	6.66	69.3	12
C7.4	-	-	-	-	-
C7.5	-	-	-	-	-

C7.6	-	-	-	-	-
C7.7	6	36.5	6.09	72.67	11.04
C8.1	12	40.5	5.87	72.25	10.61
C8.2	8	27.5	7.27	65.12	10.87
C8.3	21	31.33	11.11	67.33	16.49
C8.4	15	21.73	7.6	68.27	9.83
C8.6	-	-	-	-	-
SC1	14	37.93	4.25	72.07	8.15
SC2	10	35.9	5.59	71.4	9.28
SC4	-	-	-	-	-
SC5	-	-	-	-	-
SC7	-	-	-	-	-
SC9	8	30.25	6.8	71.75	9.9
SC10	-	-	-	-	-
C3.9	6	-	-	72.67	16.74
C5.4	18	-	-	72.67	11.44
C6.5	12	-	-	69.58	5.99
CCD	95	-	-	73.72	8.7
COD	-	-	-	-	-

The tables that follow give the question statistics for each paper for Mathematics candidates. Data for papers with fewer than six candidates are not included.

Paper C1.3: Analytic Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.83	13.83	6.40	6	0
Q2	14.4	14.4	5.89	10	0
Q3	14.4	14.4	5.02	10	0

Paper C1.4: Axiomatic Set Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.63	11.63	5.21	8	0
Q2	11.50	11.50	4.12	4	0
Q3	11.33	11.33	2.52	3	0

Paper C2.1: Lie Algebras

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.83	20.83	4.553	18	0
Q2	13.08	13.08	5.33	13	0
Q3	18	18	8.49	5	0

Paper C2.2: Homological Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.15	18.15	3.41	20	0
Q2	14	14	5.94	12	0
Q3	13.14	13.14	4.81	7	0

Paper C2.3: Representation Theory of Semisimple Lie Algebras

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.5	18.5	4.95	2	0
Q2	14	14	6.63	7	0
Q3	16.33	16.33	5.98	9	0

Paper C2.4: Infinite Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.6	16.6	5.72	10	0
Q2	13.67	13.67	4.09	9	0
Q3	14.33	14.33	2.31	3	0

Paper C2.5: Non-Commutative Rings

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	10.43	10.43	4.12	7	0
Q2	13.2	13.2	2.17	5	0
Q3	15.5	15.5	4.95	2	0

Paper C2.6: Introduction to Schemes

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	20.38	20.38	4.31	8	0
Q2	16.67	16.67	5.77	3	0
Q3	18.71	18.71	5.09	7	0

Paper C2.7: Category Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.41	15.41	3.92	27	0
Q2	14.92	15.4	5.43	25	1
Q3	14	14	1.83	4	0

Paper C3.1: Algebraic Topology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.53	15.53	7.17	15	0
Q2	14.2	14.2	5.72	5	0
Q3	16.3	16.3	6.68	10	0

Paper C3.2: Geometric Group Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.33	17.33	4.64	9	0
Q2	14.63	14.63	3.81	8	0
Q3	6	6	-	1	0

Paper C3.3: Differentiable Manifolds

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.14	15.14	3.34	7	0
Q2	12.83	12.83	3.76	6	0
Q3	12	12	-	1	0

Paper C3.4: Algebraic Geometry

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18	18	5.50	11	0
Q2	17	18.57	8.75	7	1
Q3	20.83	20.83	3.66	6	0

Paper C3.5: Lie Groups

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.63	17.63	3.66	8	0
Q2	10	10	6.93	3	0
Q3	14	14	2	5	0

Paper C3.7: Elliptic Curves

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.33	19.33	5.20	9	0
Q2	17.82	17.82	3.63	11	0
Q3	18.22	18.22	5.75	14	0

Paper C3.8: Analytic Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.38	15.38	6.08	16	0
Q2	20.2	20.2	5.73	10	0
Q3	15.17	15.17	7.41	12	0

Paper C3.10: Additive and Combinatorial Number Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.83	13.83	9.56	6	0
Q2	12	12	7.448	4	0
Q3	22	22	-	1	0

Paper C4.1: Further Functional Analysis

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.6	16.6	4.67	5	0
Q2	15.75	15.75	2.5	4	0
Q3	16.43	16.43	5.53	7	0

Paper C4.3: Functional Analytical Methods for PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.67	16.67	2.67	12	0
Q2	18.57	18.57	4.04	7	0
Q3	13.6	13.6	5.86	5	0

Paper C4.6: Fixed Point Methods for Nonlinear PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17	17	3.58	6	0
Q2	20.33	20.33	3.204	6	0
Q3	18.4	18.4	2.80	5	0

Paper C5.1: Solid Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18	18	3.82	14	0
Q2	15.85	15.85	5.76	13	0
Q3	23	23	-	1	0

Paper C5.2: Elasticity and Plasticity

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.5	11.5	3.68	12	0
Q2	11.92	12.73	4.70	11	1
Q3	15.78	15.78	3.38	9	0

Paper C5.5: Perturbation Methods

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.2	16.2	4.21	25	0
Q2	13.91	16	7.67	8	3
Q3	17.45	17.45	4.10	29	0

Paper C5.6: Applied Complex Variables

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.9	18.9	5.0	10	0
Q2	11.33	11.33	6.35	3	0
Q3	22.44	22.44	3.32	9	0

Paper C5.7: Topics in Fluid Mechanics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	11.44	11.44	2.94	16	0
Q2	17.17	17.17	4.26	12	0
Q3	18.25	18.25	3.85	8	0

Paper C5.9: Mathematical Mechanical Biology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.2	18.2	4.44	5	0
Q2	17.86	17.86	4.63	7	0
Q3	19.75	19.75	5.38	4	0

Paper C5.11: Mathematical Geoscience

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	16.75	16.75	4.18	12	0
Q2	16.37	16.37	4.46	19	0
Q3	17.58	17.58	3.39	19	0

Paper C5.12: Mathematical Physiology

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17	17	3.45	19	0
Q2	17.14	17.14	3.18	14	0
Q3	16.67	16.67	4.86	15	0

Paper C6.1: Numerical Linear Algebra

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.86	13.86	5.29	14	0
Q2	13	13	5.78	8	0
Q3	12.83	12.83	4.97	12	0

Paper C6.2: Continuous Optimization

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	14.58	14.58	1.56	12	0
Q2	15.78	15.78	4.02	9	0
Q3	14.89	14.89	2.98	9	0

Paper C6.4: Finite Element Methods for Partial Differential Equations

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	17.89	17.89	5.18	9	0
Q2	16.9	16.9	3.11	10	0
Q3	12	12	-	1	0

Paper C7.7: Random Matrix Theory

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.2	19.2	3.27	5	0
Q2	17.2	17.2	3.35	5	0
Q3	18.5	18.5	2.12	2	0

Paper C8.1: Stochastic Differential Equations

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	18.67	18.67	7.77	3	0
Q2	21.67	21.67	2.06	12	0
Q3	18.89	18.89	3.26	9	0

Paper C8.2: Stochastic Analysis and PDEs

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	7	18.67	10.10	3	5
Q2	9.5	11.67	5.35	6	2
Q3	11.75	13.43	5.65	7	1

Paper C8.3: Combinatorics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	15.63	16.53	6.89	15	1
Q2	16.07	16.77	6.31	13	2
Q3	13.71	13.71	6.044	14	0

Paper C8.4: Probabilistic Combinatorics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	13.18	13.18	5.79	11	0
Q2	9.33	9.33	3.72	6	0
Q3	9.62	9.62	3.07	13	0

Paper SC1: Stochastic Models in Mathematical Genetics

Question	Mean Mark		Std Dev	Number of attempts	
	All	Used		Used	Unused
Q1	19.43	19.43	1.62	7	0
Q2	19	19	4.32	7	0
Q3	18.71	18.71	2.37	14	0

D. Recommendations for Next Year's Examiners and Teaching Committee

• **Major recommendation: replacement of the scaling algorithm used in Part C.** We **recommend** that the scaling algorithm used in Part C should be reviewed, and preferably changed (possibly even to something completely different), before next year's exams. This could be a matter for the Undergraduate Exams Working Group.

One particular issue we want to highlight is that the algorithm assigns a USM of 60 to the Maths candidate who, counting down the ranked list, is numbered the same as the number of MATHS I's and II.1's in Part B taking this paper. However, candidates getting II.2's and below no longer go on to Part C, so this means that the algorithm essentially *always* assigns a USM of 60 to the bottom Maths candidate, whatever their score – which is daft – so the examiners essentially always have to adjust the scaling map near USM 60, even if it is sensible near USM 70.

Apart from this issue, the board of examiners feels a broader disquiet with our practices on scaling: it takes hours in the examiners meeting, and often feels quite arbitrary. We agree that scaling is essential, but maybe there are better ways to do it? There may be research available on best practices for scaling exams.

One possibility would be not to go by Part B performance, but to scale a paper based on comparing average raw marks for a given paper with average raw marks for the same set of candidates on all papers, or something like this. This would have the advantage of not excluding OMMS (etc.) candidates from the scaling data, which would be desirable. Some C papers are taken by few MMath candidates but more OMMS, MMathPhys, etc., and the current algorithm yields poor results for these.

• **Length of dissertations:** these are supposed to be maximum 7500 words long. But there is a difficulty in counting words in mathematics, especially if you only have a PDF file rather than LaTeX. Markers complain of receiving a lot of very long dissertations; when markers count words by hand, these often exceed the word limit significantly. Markers were advised to allow 10% over the word limit and then stop reading.

We **recommend** that next year, the department should impose a page limit of say 45 pages on dissertations (either as a replacement to the 7500 word limit, or in addition), with a minimum font size, to exclude over-long dissertations.

• **Penalties for late submission of online exams.** The penalties in Exam Conventions for late submission of online exams were: 0-5 minutes, no penalty, and 5-20 minutes, fail mark. (Note that a fail in one paper also excludes a candidate from getting a merit or distinction.)

The examiners felt this was too harsh. We **recommend** this should be replaced by a milder graduated penalty, e.g. 6-10 minutes late deduct 1% of marks, and so on. We received many MCEs from candidates submitting 6-20 minutes late owing to internet, phone or computer problems. Candidates in these circumstances who did not submit MCEs would have been hit very hard by this rule, if the examiners had not chosen to waive it.

The examiners would have been in favour of applying a mild penalty (say deduct 1–3 USMs) to *everyone* who submits a few minutes late, regardless of whether they had a plausible excuse or not, provided this policy was advertized to candidates in advance. Such excuses are trivial to invent, and completely unverifiable. The 30 minutes 'technical time' for scanning and uploading scripts is generous. The temptation to use some technical time to work on your script, and gamble on scanning and uploading quickly, is strong.

• **On the change to distinction/merit/pass/fail system.** This year MMath Part C moved for the first time to distinction/merit/pass/fail rather than I/II.1/II.2/III/pass/fail. This change includes the new rule that candidates who fail one paper (USM < 50) are not eligible for a merit or distinction.

The examiners feel that this rule may lead to injustices, and chose to suspend it in several cases. Mathematics examining may be more hit-and-miss than some other subjects, that is, it is probably easier for a good candidate to have a bad exam in maths than in history for instance, and we do sometimes set exams which are too hard and difficult to scale. We **recommend** that this should be replaced by a more nuanced rule.

For example, maybe candidates should lose only one degree class (i.e. drop from distinction to merit in the event of a fail paper), and maybe the rule should only apply when the candidate is less than 3 USMs above the borderline (i.e. do not demote when AvUSM $>$ 72.5 or 67.5 $<$ AvUSM $<$ 69.5).

We should also make it clearer in our instructions to assessors that 50 USMs is now an important boundary, because of the failed paper rule, and ask them for their considered opinion on what raw mark should correspond to 50.

• **Comments by external examiners.** The external examiners were surprised they were involved in any scaling discussion, and felt this should have been completed locally before the meeting. They would also appreciate a bit more feedback on how their comments on draft papers are acted on.

E. papers and on individual questions

The comments which follow were submitted by the assessors, and have been reproduced with only minimal editing. Some data to be found in Section C above have been omitted.

C1.1: Model Theory

Q1. The problem was chosen by about 2/3 of the candidates, and was generally well done by those who chose it, showing a good understanding of use of the set of types realised in a model, in a new setting. The exception was 1c. The elementary 'if' direction was again done well by most, but almost no one recognised the need to quote the omitting types theorem for the 'only if' direction (or to solve it without it.)

Q2. This was the more 'algebraic' question, again chosen by just over 2/3. The majority misread the definition of 'independent', failing to consider that the elements a; b are equal and thus no elements are taken from the finite cycles. With this misunderstanding, 2b becomes false as stated. Some were able to recover, but some were unfortunately mislead into making further mistakes.

Q3. A slightly less popular problem. Parts (a) and (b) were generally well done. In part (d) the need to use Ryll-Nardjewski was clear to all, and often solved well or with small mistakes such as saying that every type has infnitely many solutions, rather than the correct and sufficient statement that at least one does. But in other cases fundamental confusions showed up, such as trying to find an automorphism taking an n-type to an m-type for distinct n,m.

C1.2: Gödel's Incompleteness Theorems

This was the second year in which exams were affected by the Covid-19 epidemic, and the first in which all teaching was done online. Everyone tried the best they could, but there is no adequate substitute for meeting face-to-face (even the Open University runs summer schools), and the conditions people were working in varied from suboptimal and demotivating to horrifying. In this context, my opinion is that every student who made it through the year as far as writing something down for the exam, deserves to be commended.

It was decided that there should be less bookwork on this year's papers, and that they should be a little harder. In some papers the marks ended up spread out across a very wide range. That was less true on this paper, however.

Now for the individual questions.

Q1. There were various different, and ingenious, solutions given to (a)(i), the simplest of which was the Gödel sentence for Peano arithmetic.

Part (ii) of 1.(a) caused a certain amount of difficulty. There is a trap for the unwary in the apparent complete symmetry between A, B and C. It is thinkable that the exact choice of A, B and C might matter; so before using a "by symmetry" argument you need to be sure that the exact choice doesn't matter. For example, if you can prove *purely from the three equivalences given* that A is provable, then it does follow that B and C are provable, because in this situation the symmetry works.

Part (b) was done well on the whole.

In parts (c) and (d), it is vital to be very aware of the difference between the statement that a formula is true in $\mathbb{N}$, and that the same formula is provable. Failure to distinguish rigorously enough resulted in some confusion.

Q2. This was a popular question, and many good answers to it were given. Even so, each of (a)(i), (ii) and (iv) found people who were willing to claim that they were provable. In some cases, it was a question of lack of care with brackets; in others, a failure to distinguish between a formula being true in some structure and being provable.

Many good solutions were given to part (b) of question 2. In part (ii), several candidates suspected (wrongly, for once) that there was an error in the paper. There isn't; the p in the formula is deliberately not modalised, and so the Fixed Point Theorem for Gödel-Löb logic cannot be used. The formula is, however, a tautology, so any tautology is a fixed point.

Q3. was less popular, and people seemed to find it harder. It is more model-theoretic than the other questions, and the point, particularly in parts (c) and (d), is to construct models of the axiom system Q in which various strange things happen. In part (b), several candidates (correctly) appealed to non-commutativity of multiplication in the ordinals.

C1.3: Analytic Topology

This was the second year in which examining was affected, and the first in which teaching was seriously affected (or even devastated), by the Covid-19 pandemic. People were working under conditions which varied from difficult and demotivating, to horrifying. In this context, anyone who struggled through a course, this one in particular, made it to the end, and sat down to the exam, deserves to be congratulated.

The exams were taken remotely and were "open-book". It was decided, in this context, that questions should contain little bookwork and not to be too easy. The consequence was that there was very high variance in the marks. Little was done correctly by all candidates, though there were some very good answers given to all of the questions.

Q1. Part (a) is essentially a proof of Urysohn's Metrisation Theorem. In part (iv) of part (a), an uncountable discrete space is an example of a metric space which cannot be embedded in a countable product of copies of the real line.

Doing (b)(i) successfully involves noticing that M. E. Rudin's proof of Stone's Theorem proves something a bit stronger than what appears to be necessary, namely that every open cover of a metric space has a σ-discrete open refinement (not just a σ-locally finite one). In part (ii), we apply part (i) to a countable sequence of open covers, namely the balls of radius $1/n$, for each n.

In part (c), some candidates claimed to prove contradictory things about the example given, such as that it was a metric space and therefore paracompact, but not T_3. It's always worth doing a reality check on anything that you've written. In fact the example is a standard-ish example of a Hausdorff, non-regular space, and it's also second countable and so certainly has a σ-discrete basis.

Q2. In (a)(ii), an uncountable product of non-trivial metric spaces is not first countable, and therefore not metrisable.

(b)(ii) posed difficulties. Many candidates spotted that they should use Urysohn's Lemma with the (complements of the) sets U_n in the previous part, thus generating a countable number of functions f_n, from which f needed to be extracted. This last step proved surprisingly difficult. It's enough to adjust the f_n so that their sum converges uniformly, and then define f to be the limit.

The easiest example of a space X for (b)(iii) is probably an uncountable discrete space with an extra point joined to it, all neighbourhoods of which include all but countably many points of the uncountable discrete set. The closed set C is then the set whose only element is this single point. The example given in the model solutions is more high-powered than this and requires knowledge of set theory.

Part (c) of Q2., when joined with the results of Q.1., gives the essentials of the proof of Bing's Metrisation Theorem, which states that a space is metrisable if and only if it is T_3 and has a σ-discrete basis. The reverse direction involves embedding the space in a countable product of hedgehogs, in a manner described in part (c) (metrisability is not actually necessary for the arguments in part (c) to work).

Q3. In (a)(iii), many candidates gave an incorrect notion of uniqueness. It's quite possible to have compactifications which are homeomorphic as spaces but not equivalent (exercise for the reader).

In part (b), (i) is a tweak of a question on one of the problem sheets. The difficulty in the first paragraph of (ii) is proving that βX has a basis of clopen sets; part (i) only gives us a large quantity of clopen sets, not a basis consisting of them.

In (c)(ii), it is necessary to show that the point one adds to a space to create the one-point compactification, has a local basis consisting of clopen sets. Many candidates failed to spot how to do this.

After the difficult arguments of the previous parts of the question, (c)(iii) is relatively straightforward: it involves putting (a)(iii), (b)(ii) and (c)(ii) together and drawing appropriate conclusions.

C1.4: Axiomatic Set Theory

This was a hard paper with candidates not reading the questions carefully and often missing the main points of the parts of the question or how different parts interacted.

Question 1 was by far the most popular question, whereas questions 2 and 3 were attempted roughly equally often.

A surprisingly large number of candidate submitted only very limited amount of work.

Question 1: In (a)(ii) a number of candidates erroneously assumed that the classes satisfy fragments of ZF (in particular the **Union**). Hardly any candidate remarked that transitivity was absolute for A,B and a large number of candidates only gave examples of A, B, x, z, z' such that $(z = TC(x))^A$, $(z' = TC(x))^B$ and $z \neq z'$ without explaining why this showed $(\neg\, z = TC(x))^B$.

In (b) a lot of the attempts for (i) and (iii) were overly long and complicated and often incomplete. For the examples in (ii) and (iv) only very few candidates used concrete F to show that certain failures of **Foundation** can indeed happen in $\mathbf{ZF}^-$.

Only very few candidates made a credible attempt at (c).

Question 2: Part (a) was mostly well done although not all candidates explicitly mentioned that C is transitive and hence a lot of formulas are absolute.

In part (b) only few candidates even attempted to describe how to modify a formula with parameters from a to a suitable formula with paramters from $a \cap x$. No candidate picked up on the importance of defining the (global) well-order inside B_a for which being able to assume (from (i)) $a \in B_a$ is crucial.

Part (c) was attempted only by very few candidates.

Question 3: Part (a)(iii) was harder than expected with only few candidates attempting this.

Because of its modular nature candidates gained a reasonable amount of marks in (b) but most applied well-foundedness of R directly to non-empty classes without explanation. Proofs for $S(x) \in \mathbf{On}$ were overly complicated with only few candidates considering an R-minimal counterexample. Only few candidates attempting (c) realized that the assumption of R being set-like had been dropped.

C2.1: Lie Algebras

This paper was generally well-answered. Question 1 was attempted by all candidates, and most scored well on it. Questions 2 and 3 were more evenly divided, with question 2 being slightly more popular. A number of candidates lost marks in question 2 because they ignored the condition that the semisimple and nilpotent parts of an element must commute with each other. Those attempting question 3 scored well, though most lost marks in the final part by failing to properly justify why a direct sum decomposition of the Lie algebra will yield a corresponding decomposition of a Cartan subalgebra.

C2.2: Homological Algebra

Q1: All candidates attempted question 1. The standard of answers was quite good. Not all candidates realised that the length of the resolution only gives an upper bound on the dimension, and that one needs to show Ext non-vanishing in order to get a lower bound. The last part of the question was challenging and very few managed it.

Q2: The standard of answers was varied. Not all candidates used the exactness of the colimit correctly in their proofs. The last part of the question was challenging and there were few attempts of it.

Q3: The standard of answers was varied. Few candidates used Kunneth in order to show that the tensor product of the bar resolutions is a resolution in part a. There were not many attempts of part c as it required a good understanding of the proof of part a. Few attempted the last part of the question.

C2.3: Representation Theory of Semisimple Lie Algebras

Question 1 was selected by fewer students. Part (b)(i) could be appropached in two rather different ways: one was to use the Weyl dimension formula to determine the dimension of the representation, thus deducing the dimension of the missing weight space. The second way was to use the inductive algorythm for computing weight multiplicities, based on the Weyl character formula. In Part (c)(i), a typical mistake was to claim that the short roots correspond to a subalgebra of G_2 (it's the long roots which correspond to a subalgebra of G_2).

Question 2 Some students made their life unnecessarily complicated in Part (a)(i) by using generalised weight vectors as opposed to weight vectors – without realising that this doesn't change the set $\Psi(V)$. The computations in Part (b)(i) look rather different depending which definition of $\mathfrak{so}(2n)$ one adopts: there exist at least three different ways of describing $\mathfrak{so}(2n)$ in terms of matrices, and the Cartan subalgebra $\mathfrak{h}$ ends up looking quite different depending on the choice.

Question 3 In Part (c)(i)(β), there was some confusion as to whether n was assumed to be an integer or not (my intention was for $n \in \mathbb{N}$ to be assumed). Luckily, as the case $n \notin \mathbb{N}$ was anyways a part of what was being asked in Part (c)(ii)(β), no harm was done to those students who spent the extra effort to describe what happens when n is not an integer. Part (c)(ii)(β) was harder than expected, and no students noticed that the module described in Part (b) has the same formal character as a Verma module.

C2.4: Infinite Groups

Question 1 was well answered, and for the last part of it several different approaches were provided. Most students went on to answer Question 2, and while displaying solid knowledge of the linear and nilpotent groups, some of them spent a lot of time on standard matrix calculations, and did not manage to get to the last question, on the order of growth. About a third of the students attempted Question 3, and provided almost complete answers. A few did not realize that the map F is not a group homomorphism and therefore a proof of its injectivity cannot reduce to a study of the kernel.

C2.5: Non-Commutative Rings

Question 1: This was the most popular question, with only one student not having a good go at it. Part (a)(i) is essentially bookwork, but was nevertheless not done well. For part (ii), people tried to use the $aRb \Rightarrow a = 0$ or $b = 0$ definition of prime ring. However, using the "product of two non-zero two-sided ideals is again non-zero" formulation is much easier here. (a)(iii) was seen on problem sheets and was done very well by most. Part (b) was OK. Part (c)(i) was the hardest part of Question 1, with only one person getting somewhere in the solution. Since this exam was open-book, recitations of the proofs of Goldie's Theorems were not required; a couple of people also managed to mis-read the question. Part (c)(ii) was mostly OK.

Question 2: (a) was fine. In (b)(i), about half of the students didn't try to compute the endomorphism ring D of $k[x_1, \cdots, x_n]$ as an $A_n(k)$-module, and consequently lost a lot of marks. (c)(i,ii) was fine but (iii) defeated nearly everyone. Question 2 was attempted by around half of the students.

Question 3: parts (a,b,c) were mostly done well. However no-one managed to solve (d) fully: the key difficulty is realising that one can reduce to the case where $\widetilde{A}/J$ has no non-zero t-torsion elements. If this condition holds, then it is possible to show that the natural filtration on the de-homogenisation $I = \cup_n J_n$ of a graded left ideal $J = \oplus_n J_n t^n$ of the Rees ring coincides with the subspace filtration on I.

C2.6 Introduction to Schemes

Almost all candidates attempted Q1, and then about twice as many attempted Q3 compared to Q2. The average in the three questions was 19, 15, 17.

In Q1(a) there was a minor typo in the exam where it asks to find the topology of X, namely there was a Hint that candidates should show that "non-empty proper Zariski closed sets of $\operatorname{Spec}\mathbb{Z}[i]$ are the finite subsets" (that part had no typo) but the following text "of $\mathbb{Z}[i]\setminus\{0\}$" should have been "of $\operatorname{Spec}\mathbb{Z}[i]\setminus\{0\}$. Fortunately, from the scripts it appears that candidates were all content with the main idea of the Hint of showing that proper closed subsets were finite, which was correctly stated.

In Q2(a) in the last part candidates erroneously picked f_i for $\mathbb{P}^n$ which did not lie in $\underline{F}^*(U_i)$.

In Q3(b) and Q3(c) not all candidates exploited the knowledge from the course about how push-forward and pull-back transform modules.

C2.7 Category Theory

Most candidates found this paper hard; although there were many good answers there were no essentially perfect solutions to Questions 1 and 3 and only one to Question 2.

Much of Question 1 was competently answered (although as in past years there were disappointingly many candidates who did not give a correct description of coequalisers in Set). However there were no serious attempts at the last part 1(b)(iv); this had been expected to be found difficult and had not been allocated many marks.

A good number of candidates provided first class solutions to Question 2. Some candidates were tripped up in 2(a)(ii) by forgetting that a pointed set must be nonempty; most found it hard to produce a counterexample for 2(c)(ii), although they believed correctly that the statement was false. There were some nice solutions to 2(d).

Question 3 was far less popular than the other two, and only one solution to this question was given a first class mark.

C3.1: Algebraic Topology

The quality of the scripts was extraordinarily varied, with 28 scripts and roughly 3 candidates within every 5-point raw-mark-bracket between 0-50. Four scripts were essentially perfect, but the average scores on the three questions were rather low: 15, 13, 14. Almost all candidates attempted Q1, but almost twice as many preferred Q3 over Q2. Generally, it seemed candidates had not absorbed many ideas from the course, in contrast to the previous year.

Question 1.
There appeared to be fundamental knowledge gaps in many scripts, for example (1) candidates not appreciating the word ‘free’ in Q1(a) therefore omitting half of the bookwork proof, and many candidates not being able to reproduce the other half of the proof with precision; (2) many candidates were splitting sequences without justification in Q1 despite explicit Lemmas in the course notes highlighting when it is allowed, and numerous proofs in the notes repeatedly justifying tricks to split. A suspicious number of scripts tried to solve Q1(b) by mentioning a cycle basis (or fundamental cycles) from graph theory, most without proof and, crucially, not justifying why the homology groups were free (despite emphasis in the course on submodules of free modules being free, in particular the kernel and image of maps between free abelian groups being therefore free). In Q1(c) the Hint was one step away from imploring candidates to use Alexander duality; most candidates steamed ahead with many pages of excision calculations that led nowhere.

Question 2.
In Q2(b) candidates often did not justify why it was a chain map, and wrote down a functori-

ality diagram without proving that it commuted. In Q2(c) most candidates underestimated the proof of exactness which, although not hard, is not as easy to justify as might first be expected. No candidate did the very last part of Q2(d) correctly, due to a slip: on homology with $\mathbb{Z}/2$ coefficients, in the top degree, candidates assumed incorrectly that the projection $S^n \to \mathbb{RP}^n$ was non-zero.

Question 3.
In Q3(a) several candidates muddled up how they presented the ring due to an issue in degree zero: if one presents the cohomology as a direct sum of the cohomologies of the two spaces, then in degree zero the respective units $(1, 0)$, $(0, 1)$ would incorrectly multiply to zero (even if one declares them to be identified). Many candidates struggled with homology calculations in Q3(b) and Q3(c), despite being given two spaces which up to homotopy equivalence are a point and a circle respectively (the two simplest spaces available, after which the next simplest are: Q1 about collections of circles and Q2 about spheres). Several candidates thought the space in Q3(b) was one infinite cone with a vertex, rather than two infinite cones joined at the vertex, perhaps being thrown off by the space being declared as a 'cone'. This of course made it harder to justify why the space was not a manifold, since in that case it is a topological manifold, but candidates nevertheless managed to find a reason.

C3.2 Geometric Group Theory

Question 1 was attempted by all candidates, with good results on the whole. Most of the mistakes were in part c, where a number of attempts to change the presentations were made without any kind of method, and this turned out to be either unsuccessful or time consuming.

Question 2 was likewise popular. In the second part of (a) a number of candidates tried to use properties of universalities of amalgamated products, instead of an approach using actions on Bass-Serre trees. Surprisingly, the last question had the least number of successful attempts.

Question 3 was attempted only by three candidates out of 20,with only about a quarter of the questions receiving correct answers. This may be as usual related to the fact that this question covered the last part of the course.

C3.3: Differentiable Manifolds

Students obviously found this exam challenging and so I would definitely recommend some significant scaling. There were very few easy marks to be gained (due to the lack of bookwork) which meant that all marks were shifted down as a result. As an indication, the top raw mark was 37/50 (clearly by a strong student), two students achieved the lowest raw mark which was 7/50 and the average raw mark was approximately 23/50. All questions appeared to be roughly equally challenging based on student performance.

No changes were made to the marking scheme.

If possible, please give your own estimate of which raw mark out of 50 should correspond to a scaled mark of 70 (out of 100, the bottom of the First class), which raw mark should correspond to a scaled mark of 60 (the bottom of the Upper Seconds) and which raw mark should correspond to a scaled mark of 50 (the bottom of the Lower Seconds).

C3.4: Algebraic Geometry

Question 1: This question was attempted by the overwhelming majority of candidates. Some students failed to see in (c) why the ideal is not prime, but most solutions cleared that hurdle. In (d), most students found the right form of the map, but several attempts at proving the surjectivity of this map were imprecise or incomplete, in particular including some handwaving around "choice of roots of unity". Some students proposed formulas for an inverse that included cube and other roots; such maps are clearly not regular in the sense the course defined regular maps. In (e), some students failed to address irreducibility; the easy argument that (d) implies that C_2 is a curve (so dim 1) was often missed. As for (f), most students failed to find the straightforward proof that the obvious 2-generator ideal is in fact prime.

Question 2: This question was attempted by about half the candidates. In (b), many students failed to notice that by differentiating x^tBx, we get that the singular locus is simply the projectivisation of $\ker B$. Lots of essentially complete solutions were given to (c), either by direct calculation, or by recalling (in full) the Segre embedding. Part (d) was also done well by many students, either by explicitly factoring the equation or by writing down the equations of some line and solving equations for the coefficients. A comment that stated simply that "this is a famous theorem about cubic surfaces" only received fractional credit, as this theorem was not part of the course.

Question 3: This question was also attempted by about half the candidates. Some students failed to note in (a) that essential aspects of a resolution of singularities are that it should be a surjective and a morphism. In (b), some students failed to address why the cover they propose is by affine varieties. (d) was generally done very well. In (e), some computational mistakes or failure to look at all affine charts suggested to some students that point blowup might actually lead to a resolution. (f) was generally done well unless time pressure prevented students from addressing this question.

C3.5: Lie Groups

Candidates found this a hard paper, though there were some good answers, especially to Question 1.

All candidates attempted Question 1. The part which caused most difficulty was 1(a)(iii): very few candidates were able to describe the irreducible representations of $O(2)$ correctly. Many forgot to include the hypothesis that G should be connected in the statement of the Maximal Torus Theorem required in 1(b), though it appeared that some realised the relevance of connectedness when answering 1(d) and then corrected their answers to 1(b).

The amount of unseen material in Question 2 was perhaps off-putting to candidates. There were some nice answers to 2(a) and 2(b) but almost no attempts at (c), possibly from lack of time.

More candidates attempted Question 3. On the whole 3(a) and 3(c) were well done, although there were not many completely satisfactory descriptions of the integrand in the Weyl integration formula. 3(b) was found much harder, especially 3(b)(iii), even though there was a similar calculation in the lecture notes.

C3.7: Elliptic Curves

Question 1: Attempted by 12/23 candidates. Parts (a) and (c) were done well. The majority but by no means all candidates also scored fairly highly on part (b), taking the expected route of using Hensel's Lemma to lift the singular point (with one candidate taking a different approach).

Question 2: Attempted by 16/23 candidates. Parts (a) and (b) were fairly routine and most scored highly on them. There were a number of different approaches taken for (c), probably the simplest being that twice the point does not satisfy the Nagell-Lutz theorem. Part (d)(i) was quite hard and only a few candidates made a decent attempt at this, but most candidates scored quite well on (d)(ii).

Question 3: Attempted by 18/23 candidates. There was quite a broad spread of marks for this question, with most candidates scoring highly on part (b), but quite a number not making progress with (a)(i) even though it was closely related to a question on the problem sheets.

C3.8 Analytic Number Theory

Overall the questions were successful. Question 1 was the most popular, but all attracted a good level of responses with a good range of marks distinguishing the stronger candidates from the weaker ones. In retrospect the changes to the open book format probably made the questions a bit too long because pure bookwork parts were replaced with things that required some thought - they should certainly be quite a bit shorter for an in-person exam. A couple of candidates seemed to suffer from time-issues, but this did not seem to be a widespread problem.

Question 1 was attempted by a majority of the candidates. The first parts were generally answered well, although a surprising number of candidates struggled with part (b) despite there being a number of examples sheet questions (and past questions) of a very similar flavour. Candidates were often inprecise (or missed some subtelties of convergence) in part (c) and part (d) was a good distinguisher for the stronger candidates.

Question 2 was generally answered well by the candidates who attempted it. Most candidates scored very well on (a) and (b) - perhaps because (a) was similar to part of a question in an exam from a few years ago. Even the candidates who struggled with the harder part (c) seemed to have the main idea, which was nice to see.

Question 3(a) attracted a number of different approaches (even though it was close to a lemma from the notes) - some candidates were careless about convergent integrals, but otherwise it was answered well. Part (b) was typically answered very well, whereas part (c) and (d) distinguished candidates. I was slightly surprised how many candidates struggled with (d), even though much of this was very close to an examples sheet question.

C3.10: Additive and Combinatorial Number Theory

Question 1 was attempted by all the 11 students. Part 1(a) surprisingly caused some difficulties for about half the students; they missed the fact that after completing the square in the exponential the sum indeed becomes a Gauss sum by a change of variables. Part 1(b) (bookwork) was done relatively well, although a few students forgot to treat the case $h = 0$ separately after squaring and changing variables. Part 1(c) was also done well. Part 1(d) caused a lot of difficulty: The claim in the hint was proven successfully by a few students, but almost none correctly utilised the hint to solve the problem. A correct solution using the hint uses the Chinese remainder theorem to reduce to prime power moduli and then performs a slight refinement of the analysis in 1(c) to show that there are many solutions modulo p satisfying the conditions of the hint. These latter two points were missed by many.

The average marks for this question were 12.7.

Question 2 was attempted by the majority of students (8/11). Part 2(a) was solved perfectly by almost all who attempted it. The same applies to 2(b), apart from a couple of slight calculation errors made there. Part 2(c) (bookwork) was generally well done, though none managed to explain the density increment step of the proof quite convincingly enough. The proof sketch requested in 2(c) consisted of a step where the exponential function is discretised to make it locally nearly constant in arithmetic progressions, and a proof that the resulting density increment in a progression must terminate in bounded time. The former task was well done, but for the latter task many claimed that an increase in density as such was enough to complete the argument, whereas one needs to note that the increase in density is uniformly lower-bounded to actually assert that claim. Question 2(d) (worth only 1 point) surprisingly caused much difficulty and was solved only by two students. Here an example of a set with a certain property was requested, and one could simply take the set of odd numbers. Questions 2(e) (parts (i) and (ii)) caused much difficulty and each part had just one perfect solution. This part was a little different from the others, and almost everyone either did not use the hint or did not realise its connection to geometric series. The only correct attempt to 2(e)(ii) used the Weyl equidistribution theorem, a theorem not used on this course but one that did the job nicely. There were some partial attempts using the hint and therefore methods learned on this course.

The average marks for this question were 12.0.

Question 3 was attempted by only two students. There was great variation in the number of marks obtained (one essentially complete solution and one attempt worth only a couple of marks). Part 3(c) seems to have been the most difficult one, based on limited evidence.

The average marks were 12.0, the same as for the much more popular Question 2.

C3.11: Riemannian Geometry

Question 1. Part (a) was bookwork and was usually done well, except some candidates lost a mark for not mentioning orientations when defining the Gauss map. Part (b) was typically done well, though common errors were in not correctly stating the Gauss equation and not discussing orientations. Part (c) proved to be challenging for students. Most students correctly used the hint and then obtained the Gauss map, but then did not do the computation correctly or efficiently to obtain the principal curvatures. Almost every student attempted this question and there was a wide spread of marks from high to low.

Question 2. Part (a) was bookwork, but marks were lost for not fully stating the Hopf–Rinow theorem and for not correctly giving a counterexample in (a)(ii). Part (b) was usually done well, with only one student losing marks through lack of justification of their arguments. Part (c) proved challenging with students not seeing the correct way to use the arguments from part (b). This was noticeably the least popular question on the paper and the marks were all in the range 10–20.

Question 3. Part (a) was bookwork and almost always answered correctly. Part (b) proved challenging with difficulties on all parts of the question. Several students spotted the use of stereographic projection form the n-sphere in (b)(i), but most did not. Most students spotted the application of Bonnet–Myers in (b)(ii), but some did not fully justify it. Part (b)(iii) proved the most difficult, with most students not recalling the hyperbolic metric. Part (c)(i) was bookwork and answered correctly. Part (c)(ii) was usually done quite well, with students typically losing marks for not justifying the application of the classification of space forms, and for not explaining fully how to apply it to reach the desired contradictions for the cases when at least one of $n, m > 1$. Part (d) was usually done well, with marks typically only lost for not fully justifying the application of Synge's theorem. Almost every student attempted this question and there was a wide spread of marks from high to low.

C4.1: Further Functional Analysis

Question 1
Part (a) was generally well done. Many candidates got started in part (b)(i), but quite a number of candidates where unable to obtain the norm estimate by using the quotient norm. Only a very small number of candidates where able to give an appropriate space X to host an example for (b)(ii), such as c_0 (which might have been suggested by looking ahead to (c)) and a full counter example was rare. Part (c) proved challenging for many, and while a reasonable number had the idea of how this should work, not very many candidates were able to supply the details.
Question 2
Part (a) was generally well done. Part (ii) and (iii) was designed to test the bookwork proof of the Banach-Alaoglu theorem, and the metrisability of the weak* topology on the unit ball of a Banach space in a somewhat different setting. Most candidates did (ii) well, but (iii) was a bit more variably answered.
In part (b)(i), many candidates realised that compactness of T can be obtained as a norm limit of finite rank operators, though the details where not always well executed. (ii) proved challenging, with only a few candidates applying the spectral theorem to the compact self-adjoint operator T^*T.
Question 3
Part (a) was very well done in part (i), and in (ii) pretty much all candidates had the right ideas, but some failed to give a convincing argument for the strict inequality $f(a) > \alpha$. Many candidates gave correct answers to (iii) — often with far more details than were re-

ally needed — but some struggled with this 2D example (and gave examples which would contradiction (b)(ii)).
Both parts of (b) proved challenging, with some candidates struggling to get started, though there were a number of completely correct answers. In (b)(i) a number of candidates didn't say how an open set V_a as in the hint could be constructed. In (b)(ii) a number of candidates weren't clear how they used results from the course so as to apply (i).
(For students using this report for exam preparation in future years, the slightly unusual wording of the hint was chosen as the result in the hint appeared on the problem sheets, and not explicitly in the lecture course).

C4.3: Functional Analytic Methods for PDEs

Q1: This question as attempted by all but one candidates. Parts (a)(i), (b)(i) and the first half of (b)(ii) were handled well. Part (a)(ii) was handled mostly reasonably well. For the second half of (b)(ii), there were a number of attempts with variable degree of success. Here, if one views X as the subspace of of weakly curl-free vector fields, then one may see that the domain may give a problem to the existence of a potential function. Part (b)(iii) was attempted by about two thirds of the candidates, and about half of those could relate B to a kind of projection operator.

Q2: This question was attempted by a bit less than two thirds of the candidates. Parts (a), (c) and (d) was handled well with minor exceptions. Part (b) was most problematic despite the fact that there was a similar problem in the problem sheets where L had no kernel. Only a small number of candidates applied correctly Rellich–Kondrachov's theorem to complete their contradiction arguments.

Q3: This question was attempted by a bit more than two fifths of the candidates. Part (a)(ii) was handled mostly well. Most candidates have some right ideas for (a)(i) and (b) but fell into various traps. For (a)(i), one needs to use a correct interpolation inequality. For (b), a number of candidates incorrectly claimed that M_a was convex. Only a small number of candidates mentioned a few words on (c), and two of them suggested correctly the form of trial functions to use in $(\star\star\star)$.

C4.6 Fixed Point Methods for Nonlinear PDEs

Question 1 was about the first part of the course regarding Brower's fixed point theorem and null Lagrangians. The question was chosen by 12 students. The solutions mostly ranged from good to almost perfect, with very few cases just above the sufficiency level.
A common mistake has been to take $B \setminus \{0\}$ (which is open and bounded but not compact) as counter-example in 1 (iv), which asked about the validity of a property for compact and connected subsets.
Point (c) (ii) was new, it needed a good handling of integration by parts and Fubini's Theorem, and a good intuition about which computations to do. It was meant to be challenging and indeed only a couple of solutions addressed it properly.

Question 2 was about Schauder's and Banach's fixed point therems and applications to nonlinear PDEs, plus an application of weak maximum principle. The question needed a good handling knowledge of preliminary material such as Sobolev embeddings and standard

properties of the Laplacian. The question was chosen by 11 students. The solutions mostly ranged from good to almost perfect, with very few cases just above the sufficiency level.
A common imprecision was not to recognise that the most general space for source term f in (2) is H^{-1}, the dual of $W_0^{1,2}$. This was not penalised too much: only one mark if the rest was properly addressed.
Very few solutions did not recognise that 2(b) was an application of weak maximum principle (and got stuck in the estimates)
Applications of Schauder's and Banach's fixed point theorems in 2(d) and 2(e) were overall well done (not everyone attempted 2(e) though); from time to time there was some imprecision about the estimates and the choice of function space. An annoying error that occurred at least a couple of time was to choose L^2 as function space for the application of the relevant fixed point theorem. This choice does not make sense for the (nonlinear) PDE; notice that in point 2 (a) the function space was set to be $X = W_0^{1,2}(\Omega)$.

Question 3 was about variational inequalities and applications to non-linear PDEs of p-Laplacian type. As in question 2, also here it was necessary to have a good handling knowledge of preliminary material such as Sobolev embeddings and standard properties of the Laplacian. The question was chosen by 8 students. Apart from one exceptional case that merely attempted the question, the solutions ranged from very good to almost perfect. Point (e) was new, it needed a good handling of the notion of convexity and minimisation. It was meant to be challenging and indeed only a couple of solutions addressed it properly. The most common error was not to realise that being a critical point (i.e. satisfying the Euler-Lagrange equations) is only a necessary but not sufficient condition for being a global minimiser.

Summary: The paper has been successful. The spread of questions attempted and the spread of marks for each question do not seem to indicate to me any major deviation from the expected patterns.

C4.8 Complex Analysis: Conformal Maps and Geometry

1. (a) Most of the students can only do $p = 2$ case. (b) Most of the students did not do well. They may know (ii) is related to the growth theorem of S class, but have difficulty in analysing the integrand. One student managed to solve both (a) and (b), but no one completed (c).

2 (a) Several students did well in this problem. (b) Most of students who attempted this problem can sort out the conformal map sending the domain to the upper half plane. But since the maps they constructed are involved, they could not provide the final answer directly. One did obtain the final result. (c) Most of the students who attempted this knew the basic strategy. The justifications may be a bit rough. (d) One student solved this problem. Some mentioned the correct theorem for solving the problem, but did not provide the value of L in the theorem.

3 This question is attempted by most of the students. The students respond well to (a) and (b). (c) Some students managed to find the correct map as suggested in the hint and solved the problem. While the rest did not get the map explicitly, thus could not correctly find the greatest annuli which separates the unit circle from the image of the segment. (d) Most of the students got the idea to solve this problem, but some has difficulty in choosing

the map. (e) A few students managed to complete the problem.

C4.9: Optimal Transport & Partial Differential Equations

The overall performance of the cohort was very good. I have the impression that the exam might have felt a bit long to them based on the fact that no one attempted all parts of a single question.

Q1. This question was taken by all the students. They perform fairly well. The basic parts were done by most of the students. Half of them had an issue connecting laws of random variables and the push forward of a measure through a map to answer properly the question about the convolution. The most complicated part was only answered by 1/2 of the students.

Q2. This question was taken by 1/3 of the students. The basic parts were well answered by them. The most advanced parts that needed a new idea were partially developed. An intuition about the meaning of transporting measures seem to be missed by the students to answer the questions faster. The performance was anyhow good overall.

Q3. This question was taken by 2/3 of the students. The bookwork and seen parts of this question were properly done by most of them. However, the newer parts were not developed by many of them in full. Conditions to apply Cauchy-Lipschitz theory for global solutions of ODE system are more than checking that the velocity field is Lipschitz continuous in the unknown variable and students tend to forget it.

C5.1: Solid Mechanics

Q1: All students tried this question and did it quite well (with an 18/25 average) probably due to the format of the question (asking the students to recover a given result is always a huge help). The first 13 marks were mostly straightforward and students showed a good understanding of the basics of nonlinear elasticity. Only a couple of students made progress in the last part that required more computational skills.

Q2: Similarly to the first question, almost all students (13/14) tried this question with a decent average but only three students fully understood the problem. Most students seem to struggle with the first part that was supposed to be straightforward (a direct computation of the stresses). Some students fail to obtain the correct response coefficients and got bogged down after that. For Question 2b, there were two possible sets of identities that could have been used, leading to slightly different results. I gave full benefit to the students and full marks for the use of either set. One of the inequalities in Q2c was difficult to manipulate and I was pleased to see a few students handling it well.

Q3: Only one student tried this question and did very well. While it looked a bit more complicated than the other ones, it was probably easier than Q2 with the proper conceptual understanding.

C5.2: Elasticity and Plasticity

Question 1 This was a relatively popular question. In part (a), many candidates insisted on rigidly following the lectures notes rather than answering the question, and many obtained erroneous formulae for K (although they were the same as for Rayleigh waves). In (b)(i), almost no-one understood the contribution of the normal stress τ_{yy} to the beam equation. In (b)(ii), several candidates didn't combine P- and S-waves to satisfy the boundary conditions (although a similar example was done on a problem sheet), and many also struggled with the basic algebraic manipulations needed to get the result. Almost no-one made significant progress with (b)(iii).

Question 2 This was also a popular question. Part (a) was similar to lectures and was done well. The stronger students were able to adapt the argument for a string from the lectures to do part (b). In part (c), very few candidates understood that the beam and obstacle curvatures must match before the contact set can expand, but the stronger candidates were still able to calculate the force F for $\delta > \kappa L^2/3$. In part (d), most candidates correctly tried to use energy conservation, but either got stuck or were stymied by not having the correct formula for F from part (c).

Question 3 This was the least popular question but attracted the best solutions. Parts (a)–(c) involved incorporating adhesion in the analysis of radially symmetric deformation of a granular medium from the lecture notes, and the relatively familiar calculations were generally handled well. In part (a), several candidates ignored the hint and pointlessly derived the required properties of the Mohr circle. In part (d), most candidates were able to use the associated flow rule to derive the given equation for u in the plastic region, but almost no-one could see how to use continuity to solve for u everywhere.

C5.3: Statistical Mechanics

The exam was generally well done, with difficulties arising mainly in the final parts of the questions.

Q1 was attempted by all students. Q1(a) was generally well done with an occasional mark dropped because the final part of question part (a) on the efficiency was not completely followed through or an occasional glitch in the algebra. Q1(b) was successfully attempted by the majority of the students, though in the last part (ie (iii)) some struggled to understand what they had to do. There were also occasional algebraic errors in b(i)-(iii).

Q2 was attempted by the majority of the students, and the results were generally good, with the challenges arising from in particular (c).

Q3 was attempted by the minority of the students. Some struggled with (b) and/or (c).

C5.5: Perturbation Methods

Q1

Overall the question was answered well. The first part of the question presented little difficulty in general though a small number of candidates stopped at the first iteration, thus failing to confirm that the first iterate was indeed the first term in an asymptotic expansion. In the second part there were numerous good solutions though justifying that the infinite number of corrections, even when summed, were still $o(1/x^m)$ distinguished the best solutions.

Q2

This was the least popular question, though it was a generalisation of a very similar problem in the lecture notes. The latter entails that students trying this question as part of revision in future years will find it more difficult than intended if the lectures no longer consider this example. The fundamental difference with the lectured example was the $\exp(xu)$ term, which could be expanded via

$$\exp(xu) = \exp(xu_0 + \epsilon xu_1 + \ldots)$$

after which the structure of the question was similar, though the complexities of dealing with the $f(x, x/\epsilon)$ term, which had no analogue in lectures, differentiated the best candidates.

Q3

This was the most popular question and it was answered well in general. Most solutions picking up all or most of marks controlled the complexity by noting that using the expression in part (a), together with the equations governing φ and A_0 in part (b), gave an extensive simplification for

$$\frac{\mathrm{d}^3 y_W}{\mathrm{d}x^3} + x^2 y_W.$$

C5.6: Applied Complex Variables

Q1: This question was attempted by most candidates and was generally done well, apart from part (d) which required more independent thought. For Part (a), a few candidates were confused by interior and exterior angles, and in a number of cases the multiplicative constant C in the mapping was left too general (or was assumed to be real). Parts (b) and (c) were managed fine by most candidates. For part (d), only a few gave a reasonably explanation of why the given quantity represents the film thickness at C. Most candidates realised they needed to do some sort of integral of the equations from (c), but many were confused by what limits to take.

Q2: This question was attempted by few candidates, perhaps reflecting the fact it looked most different from previous exam questions, although it followed a similar recipe. Parts (a) and (c) were both done well. The conversion between the limits as $Y \to \pm\infty$ and those as $z \to \infty$ and $z \to 0$ caused some difficulty in (b), and no-one really got very far with part (d). In particular, all but one attempt wrongly assumed that $H(z)$ needed to be zero.

Q3: This question was done very well on the whole, especially parts (a) and (b). For part (c) a common slip was to assume that $c > 0$ without comment, and in some cases the ordering of the logic to explain why the expression is constant was not quite right. For part (d), quite a number of candidates obtained the correct result, although the algebraic manipulations required a lot of reverse-engineering in some cases. The most common difficulties were errors in computing the residues, and having the wrong orientation of the inversion contour.

C5.7: Topics in Fluid Mechanics

Of 19 candidates, 16 did question 1, 13 did question 2, 9 did question 3. The marks ranges were: > 35: 3; 30 – 35: 7; 24 – 29: 6; < 24: 3. There was a good spread of marks, with the average being on the low side. This seems to be entirely due to question 1, which was done uniformly badly by all candidate, with a marks range of 5 – 15. A simple remedy to adjust the marks would be to add 5 or 6 to the scores for question 1.

Marking question 1 was one of the more depressing experiences of my Oxford career. Most candidates chose this question as the relatively straightforward 'banker'. As indeed it should have been, the question being entirely parallel to a homework question, with the twist of having cylindrical coordinates. It was long, but it was not this that was the problem. No candidate managed to traverse the algebraic manipulation, and every answer was littered with basic error. Something went wrong here. It would be a good idea to set this question again in several years' time.

By contrast, questions 2 and 3 were reasonably well done, with a good spread of marks. There was a mild but inoffensive typo in question 3.

C5.9: Mechanical Mathematical Biology

Q1. Part (a) was done very well by candidates attempting this question. Several candidates got tangled up in the calculations in part (b)(i), which led to poor answers to part (b)(ii). Part (b)(ii) required first observing that $\mathbf{n} \equiv N\mathbf{e}_z$, and then expressing the moment balance in component form for one relation, and combining the constitutive law for moment with the boundary condition for $\mathbf{m}$ for the condition. This caused some conceptual issues, but was done perfectly by one candidate.

Q2. This question was attempted by most candidates, and was largely done well. There were no consistent trends across scripts in where marks were being lost. Some candidates struggled with the bookwork derivations in part (a) but answered part (b) well; for others it was the reverse. In part (a)(iii), obtaining full marks required finding the appropriate length scale to balance the different terms and then showing that this indeed produced a dominant balance by considering the size of the remaining term. Part (b)(ii) was a straightforward calculation via asymptotic expansion; full marks required obtaining an integrated analytic solution and correct interpretation that decreased local surface tension leads to an increased sag.

Q3. Part (a) of this question followed quite closely an example from lectures/notes, and was done well, though there were some struggles with obtaining approximate solution in the limit $b \ll 1$. Obtaining the required form in part (b)(ii) required careful manipulation of the constitutive law and attention to relations between the different variables. Some candidates seem to have not had time to answer the final part, which was entirely conceptual, and was answered well by those who attempted it.

C5.11: Mathematical Geoscience

Q1: This question was the least popular but had (marginally) the highest average mark. Possibly some candidates were put off by the fact that more of the material was unseen, although the ideas used would have been mostly very familiar. Part (a) following the lectures was done well. For part (b), there was a large range of answers, some completely correct but others quite lacking in understanding. There were several attempts to throw in spurious factors of 4 to get the required answers, common mistakes being to not identify the equation in (i) as a local energy balance rather than a global one, and to miss the Jacobian when performing the integral in (ii). Part (c) was done quite well by many, though only one candidate had the correct final interpretation for the time of day.

Q2: Part (a) was done well by most candidates, though some arguments for why the factor depending on λ that appears in (ii) was greater than 1 were missing or weak. Part (b) caused surprisingly many difficulties given how many examples of solving such problems along characteristics we covered in classes. Many candidates got full marks, but some got rather few, with common confusions being to hold the wrong quantities fixed along a characteristic, or to not notice simplifications in eliminating characteristic variables. Part (c) continued the theme of characteristics and was harder. Despite a very similar question in 2019, none of the candidates clearly noted that the characteristics are different from those in (b), and wrongly separated the two families of characteristics by the shock rather than by the c characteristic that emerges from the origin (no-one seemed to notice the relevance of (a)(ii) which had hinted that the characteristics were different).

Q3: There was an unfortunate but obvious typo in the first equation which has $\partial p/\partial z$ instead of $\partial p/\partial x$. This was either noted and corrected, or ignored, with most candidates obtaining close to full marks on part (a). There were a few slips or confusions in the non-dimensionalisation, which was deliberately (but clearly) different from the that used in the lectures. Part (b) was mostly done well, with the main errors being to not consider $x < X$, and/or to make H discontinuous at X through an inappropriate choice of constants of integration. Part (c) proved quite challenging, with many candidates perhaps bogged down by algebra rather than thinking carefully about which integrals were needed.

C5.12: Mathematical Physiology

This paper seemed to work very well. The split between attempts on the three questions was $22 - 14 - 18$, with respective marks ranges of $8 - 22$, $11 - 23$, and $8 - 22$. The range of total marks out of 50 for the 27 scripts was $18 - 45$, with $5 \geq 40$, $13 \in [30, 39]$, and $6 \in [25, 29]$. There was the almost obligatory (but luckily inconsequential) typo iin question 1, where g_C metamorphosed into g_L, The questions were long but mostly straightforward, and seemed to work well for the online environment.

C6.1: Numerical Linear Algebra

Question 1 was answered by most candidates, perhaps natually as it starts with a gentle (though not bookwork) question on the basic topic of norms. In (a-i) some tried to use the triangle inequality for norms–which in this case does not directly give the desired result. In (b-ii) many stated only the trivial $b = 0$ case; the condition $A^T b = 0$ is much more general. In (iii) some used the QR factorisation of B, others the normal equation; both are valid approaches. (c-ii) appeared to be difficult to many, while the key idea is quite simply to connect it with the result (Courant-Fisher) stated in (i) and take special cases. (iii) appears to be connected to Courant-Fisher but it is not.

Question 2 was attempted by about half of the candidates. Some used the SVD to solve (a); this isn't the simplest approach and these attempts usually failed to specify the correct H_1, H_2. The eigenvector sign choice was missed by many in (b-i), and a large number of candidates failed to mention the presence of 2019 singular values at 0 in (b-ii). Part (c) is a question inspired by randomised linear algebra; the key is to notice Q has an orthogonal submatrix. (c-ii,iii) become straightforward once one realises this fact; otherwise, one would need properties of orthogonal projections etc. Many used the subordinate inequality $\|(I - QQ^T)A\|_F \leq \|I - QQ^T\|_F \|A\|_F$, but this is weak and insufficient for the problem. In fact it is true that $\|(I - QQ^T)A\|_F \leq \|I - QQ^T\|_2 \|A\|_F$ (or more generally $\|AB\|_F \leq \|A\|_2 \|B\|_F$, a fact briefly discussed in lecture. In any case the problem can be solved without its knowledge). Some tried to use the randomised SVD algorithm by Halko-Martinson-Tropp and its theory, but that would not be successful.

Question 3 was also attempted by about half of the candidates. (a-c) were mostly answered well, though finding an example in (b-ii) seemed difficult to some. In (c) it should be noted that the final least-squares problem is upper Hessenberg, so can be solved efficiently using Givens rotations. (d-ii) appeared to be challenging to many; some suggested the randomised least-squares solver Blendenpik, but this isn't a situation suitable for it. In (e) some noted that GMRES stops making progress after the kth iteration (which is correct) but then concluded immediately that k iterations gives the exact solution. This argument (while eventually correct) needs further justification, e.g. that GMRES always finds the solution for a nonsingular system $Ax = b$ in a finite number of steps (which needs to be proved if used).

C6.2: Continuous Optimisation

In the first question, the students struggled with the original (easy) pieces of bookwork they had to derive - namely, the upper bound on the number of iterations. Also, the students struggled with the calculation of the KKT point in part b); various incorrect values were found but some good answers too.

The students coped much better with the second and third question. The downside was that in Question 3, few students truly answered as expected in the bookwork derivations, namely, where only one equality constraint was present; this problem structure had to be used in the derivations in order to acquire the majority of the marks.

C6.3 Approximation of Functions

This exam ran smoothly, with no problems of any kind. Question 3 appeared somewhat harder than the others.

Question 1: 9 students did this, getting mostly 2.1 marks with a high score of 22 and a low of 8. Most students got few points on the final part (3).

Question 2: 15 students did this, with marks ranging from 10, 13, 14 (twice) to 21 (twice), 22, 23, and 24.

Question 3: 8 students did this. The highest mark was 19, with marks at the low end of 3, 9 (twice), 10, and 11. Surprisingly many missed the factor of L in part (c) that appears when you rescale a derivative from $[-1, 1]$ to $[-L, L]$, and few could see how to do the (pretty elementary) estimate for part (d).

C6.4: Finite Element Methods for Partial Differential Equations

Q1: This question was attempted by all but one Part C student, but by only one OMMS student. It revealed a good spread of abilities across those who attempted it. Q1(a)(i) was answered correctly by every candidate. Some candidates claimed in Q1(a)(ii) that the property was immediate, neglecting to observe that $v \in \mathcal{P}_{k+1}(K)$. Q1(a)(iii) was answered correctly by every candidate. Q1(b)(i) was mostly answered well, with occasional slips in the signs of the integration by parts formula for curl. A few candidates erroneously applied the permutation rule for the scalar triple product directly to the volume integrand, rather than to the term arising via integration by parts. Q1(b)(ii) was mostly answered well. In Q1(b)(iii), a few candidates tried to modify the weak form they had already derived rather than starting from the strong form of the problem. While this approach can work, the handling of the boundary terms arising from integration by parts is delicate. A small number of candidates did not realise that $u \in H(\text{curl}, \Omega)$ does not imply that $\nabla \cdot u \in L^2(\Omega)$, and failed to integrate by parts appropriately to shift the differential operator onto the scalar-valued test function.

Q2: This question was very popular, with every candidate attempting it. Q2(a) was generally answered very well, as it was quite similar to problems seen on problem sheets. Q2(b) served useful in revealing a candidate's level of understanding. In Q2(c)(i), some candidates proposed a nonsymmetric but correct A, instead of the (obvious) symmetric A that I had in mind. These candidates were awarded full marks, but generally ran into difficulties in handling the boundary conditions in Q2(c)(ii). Q2(c)(iii) was generally answered well, with most candidates giving sharp constants with the correct parametric dependence on β. Full marks were awarded for correct arguments leading to nonsharp bounds, so long as the dependence on β (or not) was correct. In Q2(c)(iv), few candidates used the sharper $\sqrt{C/\alpha}$ bound available for symmetric problems, with several candidates erroneously claiming that the problem was not symmetric. The first part of Q2(c)(v) was generally answered well, but the latter part of characterising the kernel was only answered correctly by a handful of candidates.

Q3: This question was attempted by all but one OMMS student, but by only one Part C student. Q3(a)(i) was generally answered poorly, with few candidates getting the boundary integrals arising from integration by parts correct, and several including a dependence on the test function v in the strong statement of the boundary condition $h(u, p, n) = 0$. Q3(a)(ii) was also answered poorly, with few candidates hitting the nail on the head: the boundary condition depends on the value of the pressure, so taking a solution (u, p) and modifying it to $u, p + c)$ for $c \in \mathbb{R}$ no longer satisfies the boundary condition. Q3(a)(iii) was generally answered well, but several candidates ran out of steam in the calculations and claimed the result without showing it. Q3(a)(iv) was related to Q3(a)(i) and was marked in a manner so as not to penalise candidates for the same mistake twice. Q3(b)(i) was mostly answered well. Q3(b)(ii) was answered excellently, with most of the candidates who attempted it giving very clear arguments. Q3(b)(iii) appears to have been much more challenging, with only the best students answering it correctly. Several of these offered a beautiful argument based on applying Babuška's theorem.

C7.4: Introduction to Quantum Information

Question 1. It was the least popular question. Some students struggled with visualising mixed states in terms of Bloch vectors and failed to draw the required mixture of Z eigenstates in (a) and the mixture of Pauli states in (b). Those who failed to sketch the octahedron in part (b) had difficulties with completing part (e), which is related to (b). Only few students attempted parts (e) and (f). Those who attempted part (f) showed no difficulty in stepping through the quantum circuit. However, many students simply stated that the T gate is needed for universality without backing it up with any arguments.

Question 2. It was by far the most popular, with nearly all students having it used as a successfully attempted question. In general, it was very well answered, and students scored well. Part (a) was bookwork; part (b) was done in a few different ways, but almost always successfully; part (c) was where most students dropped a few marks, having calculated the required identity successfully, but then incorrectly assuming that they could simply square this to obtain the probability; part (d) was well answered, but many students simply listed four probabilities without even mentioning how they related to the actual question asked; part (e) was usually answered correctly, and many even mentioned how the maximally mixed state attains the maximal probability for this test; part (f) was usually either answered entirely correctly, or entirely incorrectly, with some students simply stating that the probability obtained in part (d) was independent of the random number generator.

Question 3. The first three parts of this question (a, b and c) were, in general, well answered. Students knew how to take partial traces, construct the Choi matrix, and how to check positivity and complete positivity of linear maps. Some students did not realise that in part (b) the reduced density operators must have the same spectrum. The last two parts (d and e) — which required visualising the action of the depolarising map on the Bloch vector — turned out to be the most challenging; students tried different approaches but only two of them (out of 27) got it right.

C7.5: General Relativity I

Overall the exam was clearly difficult, with only a small number of exceptional students able to complete two questions with only minor errors, and a larger number of students than expected struggling with many aspects of the problems.

Question 1 was relatively popular, with a large majority of students attempting this question. The first part of the question and some portion of the middle of the question was successfully completed by many students, but there were a significant number of errors of understanding. It was fairly common, for example, to believe that a stationary observer follows a geodesic. Many students were able to obtain an expression for the time difference between the two observers, however in a large number of cases this expression included constants of motion which could (and should) have been eliminated. There were several ways to do this: the simplest was to use the stability of the orbit as in the lecture notes, but this was missed by a majority of students. Consequently they were unable to finish the question.

Question 2 was the most popular question, and part (a) was successfully completed by most students. Mistakes in part (b) were more common, with some students unable to correctly vary the action. Even fewer students succeeded in part (c): most students did not begin by finding the conserved quantities given by the Lagrangian, and many students mistakenly believed that the particle moves on a geodesic, despite deriving the force in part (b). Many students were more successful with the tensor algebra bit of part (d), although the majority of students did not realise that the energy-momentum tensor they had derived was anisotropic, and since the spacetime is isotropic this requires some extra matter.

Question 3 was the least popular question. Part (a) was done successfully by the majority of students, and a good number of students did well in part (b) too, although some struggled with the tensor algebra. Many also struggled with the tensor algebra in part (c). In part (d), no student gave a satisfactory argument for $\sigma^2 \geq 0$, although some did notice that such an argument was needed. The interpretation of the result in part (e) was also mistaken in a large number of solutions.

This was a difficult exam in a year that has been very challenging for many students. Nevertheless I hoped for better results, especially with finding conserved quantities and solving equations of motion from Lagrangians — a skill which is common to many courses in theoretical physics.

C7.6: General Relativity II

Question 1: The first question was attempted by nearly all students. The tensor manipulations in part a) did not prove difficult and also part b) was executed nearly flawlessly. However, part c) was already more difficult and although most students scored a good number of points, only a few carried it out more or less correctly. Question d) i) was the most difficult part and was not answered correctly by any student. Part ii), however, was again easier with a few students scoring full marks and most students at least some.

Question 2: This question was the least popular with only a few students attempting it. Part a) was carried out well by everyone and in part b) everyone showed the equivalence of the gravitational perturbations under the gauge transformation, however, only half the students showed that in general the new perturbation is not in wave gauge. Part c) and d) proved difficult and no student produced a complete answer here.

Question 3: This question was again attempted by nearly all students. Part a) was executed flawlessly by nearly everyone. Part b) was solved very well by most students and various computational routes to the correct solution were presented. The last part was the most difficult, and while most students gained a few points here, no one delivered a complete solution. In particular students struggled with keeping track of the coordinate ranges and with constructing the maximal analytic extension.

C7.7: Random Matrix Theory

Question 1 was attempted by most of the candidates. Parts (a) and (b) were straightforward and in general answered well. Part (c) was answered correctly by most candidates, but a few attempted to evaluate the moments using contour integration, which is difficult and wasn't necessary. Part (d) was also answered correctly by many candidates, but some failed to justify which of the two solutions to the quadratic equation was the relevant one in this problem.

Question 2 was attempted by the majority of the candidates. Part (a) was straightforward. Many candidates did well on part (b), but some attempted a circuitous route. Parts (c) and (d) were in general answered correctly. Part (e) was answered well by many, but some candidates failed to justify which of the two solutions to the quadratic equation was the relevant one in this problem. Many candidates found part (e) difficult. Only a few realised that what was required was to invert the Stieltjes transform, and even fewer did this correctly.

Question 3 was attempted by fewer candidates, but most of those who did attempt it gained high marks. Part (a) was straightforward. Most found parts (b) and (c) straightforward too. Several candidates did part (d) well, but some failed to state Gaudin's Lemma or to apply it correctly. Many candidates found part (e) difficult. Only a few related $\mathbb{E}N(\alpha, \beta; A)$ correctly to the two-point correlation function, with many failing to apply the necessary scaling with n.

C8.1: Stochastic Differential Equations

Question 1 was approximately as popular as Question 3, and less popular than Question 2. The induction argument used in Part (a.ii) presented some difficulties, and most students solved Part (a.iii) using Levy's theorem. The primary difficulty in Part (b) was that the process X_t is not a martingale, and therefore statements made using the Burkholder-Davis-Gundy inequality (for example, in Part (b.iii)) must first be proven for the martingale Y_t and then deduced for X_t as a consequence. In Part (c) it was acceptable to simply state that the process Y_t is Gaussian, but this can also be deduced by the Gaussianity of Brownian motion and the Dambis-Dubins-Schwarz theorem using the quadratic variation of Y_t calculated in Part (b). It then follows that X_t is a mean-zero Gaussian random variable with variance calculated in Part (b), from which the claim follows.

Question 2 was the most popular question of the exam. Most students proved uniqueness in Part (a) by identifying the solution explicitly, although some cited major theorems proven in the course. Parts (b.i) and (b.ii) were universally solved using a random time change and the Girsanov theorem. Part (b.iii) was more difficult, and required that by uniqueness in law the two equations could be solved on a single probability space using an absolutely continuous change of measure, where the absolute continuity is a consequence of the positivity of the stochastic exponential. Most students solved (c.i) using the Dambis-Dubins-Schwarz theorem and properties of Brownian motion, and then argued in several different ways by contradiction to prove (c.ii).

Question 3 was approximately as popular as Question 1, and less popular than Question 2. Most students solved Part (a.i) by applying Itô's formula. Part (a.ii) presented more difficulties, and the fastest solution was to take the expectation of the square of the stopped process and argue using Fatou's lemma and continuity. Part (a.iii) was a consequence of the occupation formula. It is possible to prove Part (b) directly using only the occupation formula. However, many students ended up reproving a more general statement from the course to prove Part (b.ii) in particular. Part (c.i) is a consequence of Levy's theorem. Part (c.ii) is easier if $a = 0$. If $a \neq 0$, to construct a second strong solution some students reflected the process at the first time it hit a.

C8.2: Stochastic Analysis and PDEs

All three questions were approximately equally popular. Most candidates managed to get most or all points on part a of the three questions; similarly for part b most candidates made substantial progress; a couple of candidates made very good progress on 1c but on 2c and 3c, very few candidates made substantial progress. A common mistake with 1b was to miss to argue that these are all the eigenvalues. For 2c very few manged to exploit that by using $\mathbb{Q}$, the density of Z becomes easier to handle. Similarly, most candidates made progress on 3b(i) although fewer managed to realize that 3b(i) allows to substantially simplify 3b(ii). Very few made any progress on 3c which could be done quickly by using $P_{a,b}$, the Markov property, and continuity in b.

C8.3: Combinatorics

Q1: Most candidates completed the first two parts successfully. Part (c) was most commonly attacked by using Local LYM to adjust the set system; not many candidates spotted the quicker argument using random maximal chains. Part (e) was not completed successfully by many candidates. A number of candidates assumed that $\mathcal{A}$ and $\mathcal{B}$ were antichains, which was not in the question.

Q2: The first three parts were generally done well. The last part was harder, although there were some ingenioous attempts. One or two candidates gave a construction and claimed it was the 'worst case', rather than proving the upper bound.

Q3: Candidates mostly did well on part (a), which started with a couple of variants on results from lectures. Part (b) was found more difficult. Some candidates used the Sauer-Shelah theorem without stating it properly (as the instructions below the question requested).

C8.4: Probabilistic Combinatorics

Question 1, which should be quite straightforward (though a little long), was surprisingly poorly done. Sometimes the problem was trying to apply the first and second moment methods to the wrong variable (I_n, rather than the number of increasing runs of a suitable length k; analagous to trying to calculate the expectation of the clique number rather than, as in the notes, the expected number of cliques of a suitable size). Sometimes it was figuring out when the expectation tends to 0/infinity. And sometimes just general confusion. There were some good answers though, so the question generated a wide spread of marks.

Question 2 was the least popular. With hindsight, the start is rather difficult, since it involves a slightly messy calculation. Still, most of the marks were available for a version ignoring rounding, which is then pretty simple. The start of part (b) is also a bit tricky, but the rest (simple rescaling) should be very easy. And part (c) is really going through arguments in the notes (the exponential tail bounds on component sizes, proved via random walks) with only minor changes (using the random walk to analyze the branching process itself directly). A few candidates saw this, but not many. There were several decent answers to part (d).

Question 3 was very disappointing. Although it was the most popular, it also had the lowest average mark! The first part should a totally routine application of Janson, but almost all attempts contained at least minor mistakes (e.g., giving incorrect formulae for the number of possible 5-cycles, although this doesn't affect the result). Very few candidates clearly explained the possible intersection patterns (or the key fact, all that is needed, that the intersection of two distinct 5-cycles contains more vertices than edges). (b)(i) was intended to imitate the colouring argument from the final chapter of the course; some candidates understood this, but not many (perhaps not everyone made it that far through the lecture videos). Part (c) caused a lot of trouble, with no really good answers. The key is to condition on the neighbourhood of v, regarding that as fixed, and then consider all paths of the right length starting and ending in the neighbourhood of v, and simply to apply Janson to this random variable. (Other methods work too!)

C8.6: Limit Theorems and Large Deviations in Probability

Question 1. 6 out of 10 candidates attempted this question which turns out the most challenging question of the paper. Most of candidates know well the concepts of tightness, the definition and some criterions for relative compactness, but the candidates find difficulty to apply these criterions to concrete situations.

Question 2. 7 candidates attempted this question. Most candidates are able to prove the triangle inequality for the metric on the probability measure space, and answered well about the relation with the weak convergence. Part (b) even with the hint proves difficulty, and no good solution is seen. While most candidates give good solutions to part (c).

Question 3. Again 7 candidates did this question about large deviation principle. Most candidates did quite well about the application of Cramér's LDP to evaluate some limit probabilities associated with random walks. Also most candidates have no problem to apply LDP for Brownian motion to obtain LDP for Wiener functionals. It turns out that the candidates scored better than other questions.

Statistics Units

Reports on the following courses may be found in the Mathematics and Statistics examiners' report.

SC1 - Stochastic Models in Mathematical Genetics SC2 - Probability and Statistics for Network Analysis SC4 - Advanced Topics in Statistical Machine Learning SC5 - Advanced Simulation Methods SC7 - Bayes Methods SC9 - Interacting Particle Systems SC10- Algorithmic Foundations of Learning

Computer Science

Reports on the following courses may be found in the Mathematics and Computer Science examiners' report.

Quantum Computer Science Categories, Proofs and Processes Computer Animation

F. Names of members of the Board of Examiners

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  Prof. Harald Oberhauser
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